Coordinate Geometry and inequalities   4024
Compiled by :Mustafa Asif

27. Coordinate geometry	• demonstrate familiarity with Cartesian coordinates in two dimensions • find the gradient of a straight line • calculate the gradient of a straight line from the coordinates of two points on it • calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points • interpret and obtain the equation of a straight line graph in the form $y = mx + c$ • determine the equation of a straight line parallel to a given line • find the gradient of parallel and perpendicular lines	e.g. find the equation of a line parallel to $y = 4x - 1$ that passes through (0, −3) e.g. find the gradient of a line perpendicular to $y = 3x + 1$ e.g. find the equation of a line perpendicular to one passing through the coordinates (1, 3) and (−2, −9)
20. Solutions of equations and inequalities	• solve simple linear equations in one unknown • solve fractional equations with numerical and linear algebraic denominators • solve simultaneous linear equations in two unknowns • solve quadratic equations by factorisation, completing the square or by use of the formula • solve simple linear inequalities	Includes writing a quadratic expression in completed square form.
21. Graphical representation of inequalities	• represent linear inequalities graphically	Linear programming problems are not included.

Videos for understanding

https://www.youtube.com/watch?v=PXnAKcBipKM

https://www.youtube.com/watch?v=XxZDV2K3oIs

https://www.youtube.com/watch?v=iQ8Mrq9Dp1w

https://www.youtube.com/watch?v=UfaOr5jvxSE

https://www.youtube.com/channel/UC13hWQDKJsB7IXfEl9mG4wA

Coordinate Geometry and inequalities 4024
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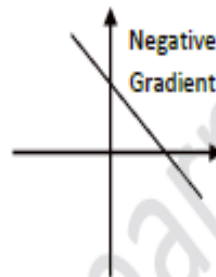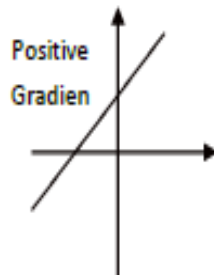<u>**Key Points**</u>

- <u>**Distance Formula**</u>

The distance between two points P $(-x_1, y_1)$ and Q $(x_2 y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- <u>**Gradient of Straight Line (m)**</u>
- Gradient $= \dfrac{rise}{run}$
- The gradient of the line joining $P(x_1, y_1)$ and $Q(y_2, y_2)$ is $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Positive
Gradien

Negative
Gradient

- <u>**Perpendicular Lines**</u>
- For perpendicular lines: $m_1 m_2 = -1$
- If the gradient of a line is m then the gradient of its normal is $-\dfrac{1}{m}$

- <u>**Parallel Lines**</u>
- Parallel lines have equal gradients. $\quad m_1 = m_2$

- <u>**Collinear Points**</u>
- Collinear points are points that lie on the same straight line.
- Let there are three point A, B and C lie on the same straight line then gradient between point A and point B is equal to the gradient between point B and point C.

- <u>**Mid Point formula**</u> Mid-point between (x_1, y_1) & (x_2, y_2) is

$$\left[\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right]$$

- <u>**Equation Of Straight Line:**</u> $\qquad y = mx + c$

- The equation of a straight line which cuts off intercepts a and b on the x-axis and y-axis is

$$\dfrac{x}{a} + \dfrac{y}{b} = 1$$

- Equation of a straight line parallel to the y-axis at a distance 'a' from it is x=a.
- Equation of a straight line parallel to the x-axis at a distance 'b' from it is y=b.
- Equation of x-axis is y=0
- Equation of y-axis is x=0.
- The equation of a straight line passing through the origin (0,0) is y=m x.

➢ **Area of a triangle**

- Draw the triangle by using the given coordinates and calculate the area of triangle by applying the formula: area of triangle $= \frac{1}{2} \times h \times b$,
  where h= height of triangle of triangle and b= base of triangle.

OR

- The area of the triangle formed by the three points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

OR

$$x_1 \qquad x_2 \qquad x_3$$
$$y_1 \qquad y_2 \qquad y_3$$

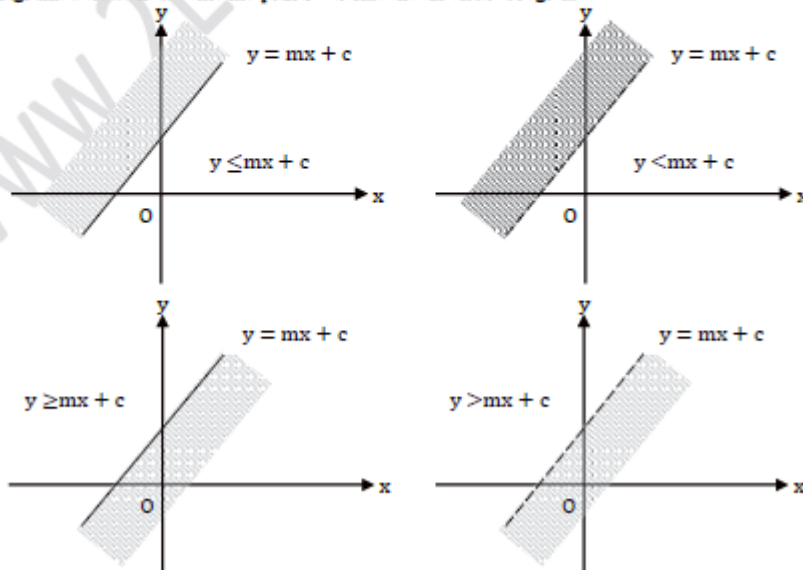$$\text{Area} = \frac{x_1y_2 + x_2y_3 + x_3y_1 - x_3y_2 - x_2y_1 - x_1y_3}{2}$$

Unit-12

Inequalities

Key Points

Graphical Representation of Linear Inequalities

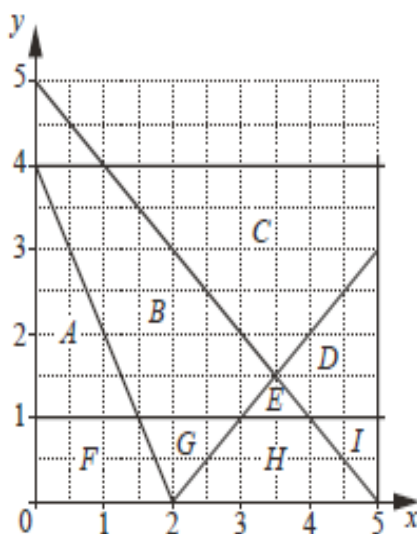➢ The diagrams below show some inequalities define the un-shaded region.

1   The diagram shows the regions A to I.   SP18/01/12

Give the letter of the region defined by each set of inequalities.

(a) $x>0, y>0, y<1$ and $y<4-2x$

Answer .... [1]

(b) $y>1, y<x-2$ and $y<5-x$

Answer .... [1]

1(a)	F	1	
1(b)	E	1	

2   A line segment, AB, joins A (3, 2) to B (−1, 10).

m/j19/11/17

   (a) Find the coordinates of the midpoint of AB.

(................ , ...) [1]

   (b) Find the equation of the perpendicular bisector of AB.

.... [4]

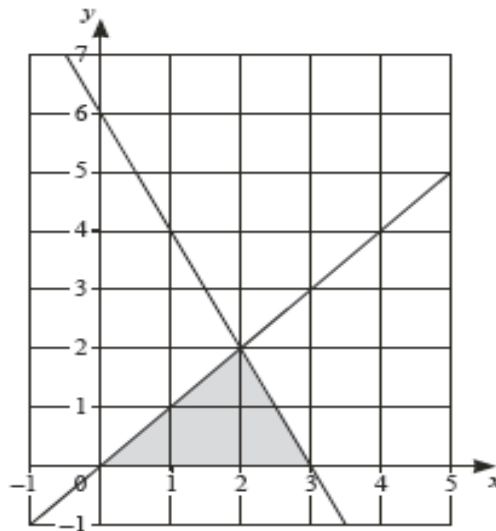| 2(a)	(1, 6)	1	
2(b)	$2y = x + 11$ oe	4	**M1** for $\dfrac{10-2}{-1-3}$ oe or gradient -2 **M1** for gradient of perpendicular $= \dfrac{1}{2}$ FT *their* gradient **M1** for attempt to find c in $y = mx + c$ using (1, 6) or FT *their* **(a)** and gradient

3   (a)   (i)   Solve the inequality $10 < 3(x+1) \leqslant 24$.   M/J19/21/6

.... [3]

(ii)   State the number of integers, x, satisfying $10 < 3(x+1) \leqslant 24$.

.... [1]

(b)

Find the 3 inequalities which define the region shaded in the diagram.

.... [3]

3(a)(i)	$\dfrac{7}{3} < x \leqslant 7$ oe final answer	3	**B2** for $\dfrac{7}{3} < x \leqslant 7$ seen or final answer $\dfrac{7}{3} < x$ or final answer $x \leqslant 7$ **B1** for either correct inequality seen or **M1** for $\dfrac{10}{3} \, [\ldots] \, x + 1 \, [\ldots]$ $\dfrac{24}{3}$ or $10 - 3 \, [\ldots] \, 3x \, [\ldots] \, 24 - 3$ soi
3(a)(ii)	5	1	
3(b)	$y \geqslant 0$ $x \geqslant y$ oe $2x + y \leqslant 6$ oe	3	**B1** for each inequality If 0 scored, **SC1** for 3 correct equations soi

4   (a) Represent the inequality $-3 < x \leqslant 2$ on the number line

below.   m/j19/22/3

[1]

(b)
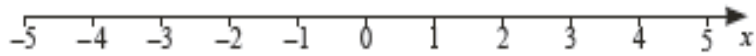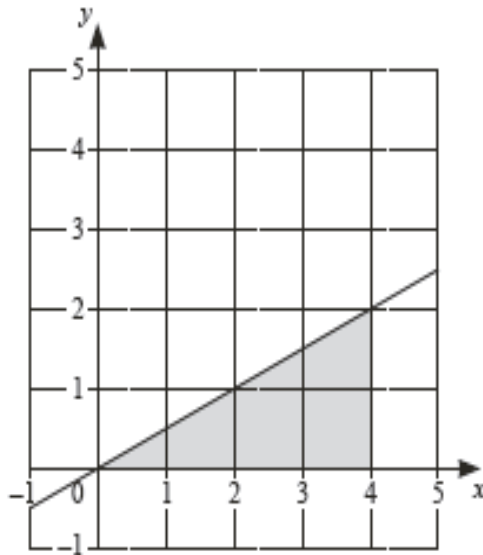
Find the 3 inequalities which define the region **shaded** in the diagram.
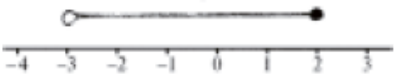
....

....

.... [3]

(c) Solve $-12 \leqslant 4(m-2) < 10$.

.... [3]

4(a)		1	
4(b)	$y \geqslant 0$ $x \leqslant 4$ oe $y \leqslant \frac{1}{2}x$ oe	3	B1 for each inequality After 0, SC1 for 3 correct equations soi
4(c)	$-1 \leqslant m < 4.5$ final answer	3	B2 for $-1 \leqslant m < 4.5$ seen or final answer $-1 \leqslant m$ or final answer $m < 4.5$ or M1 for $-\dfrac{12}{4}$ [...] $m-2$ [...] $\dfrac{10}{4}$ or $-12 + 8$ [...] $4m$ [...] $10 + 8$

5

O/N18/11/21
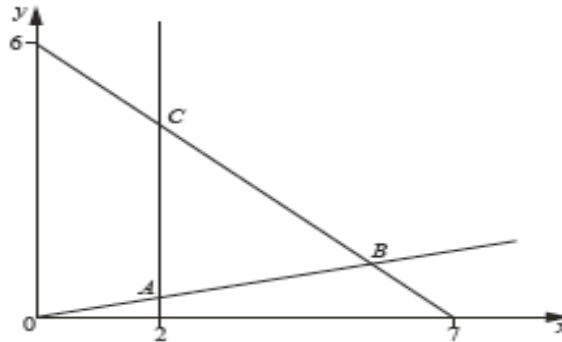
In the diagram, the equation of the line

- through B and C is $6x + 7y = 42$
- through A and B is $y = \frac{x}{5}$.

(a) The region inside triangle ABC is defined by three inequalities.
One of these is $y > \frac{x}{5}$.

Write down the other two inequalities.

Answer ....

.... [2]

(b) The line $y = kx$ passes through triangle ABC.

Find all the possible **integer** values of k.

Answer .... [2]

5(a)	$x > 2$ oe **and** $6x + 7y < 42$ oe	2	**B1** for one correct or for x ... 2 oe **and** $6x + 7y$ ... 42 oe, with incorrect (in)equalities for ... .
5(b)	Both 1 **and** 2, only, nfww	2	**B1** for C is (2, 4. ...) oe; or for gradient of $OC = 2$. ... oe

6   P is the point $(-3, 4)$, Q is the point $(5, 1)$.

(a) M is the midpoint of PQ.　　　　　　　O/N18/12/20

Find the coordinates of M.

Answer   (................. , ...) [1]

(b)   Find the gradient of PQ.

Answer   .... [1]

(c) R is the point $(-6, 0)$, O is the point $(0, 0)$.

Which of the points, R or P, is closer to O?
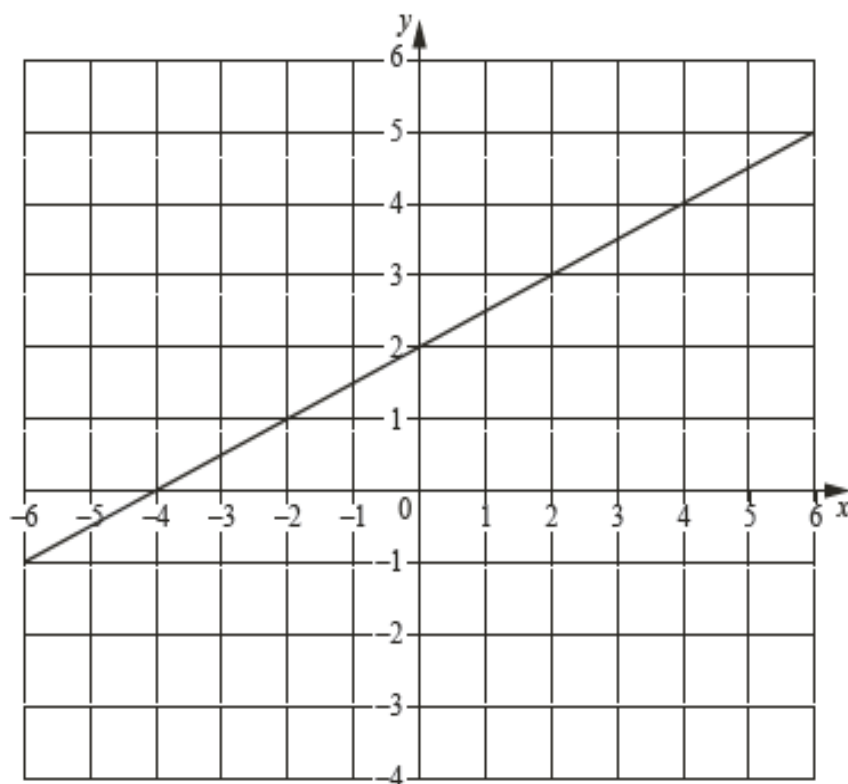Show your working.

Answer point .... [2]

6(a)	$(1, 2\frac{1}{2})$	1	
6(b)	$-\dfrac{3}{8}$ oe	1	
6(c)	P, with supporting evidence, nfww e.g. $OP = 5$, $OR = 6$	2	**B1** for $OR = 6$ nfww or **M1** for $\sqrt{(-3)^2 + 4^2}$, or better

7   O/N18/22/8

The line $2y = x + 4$ is drawn on the grid.

(a) (i) On the grid, draw the line $x + y + 2 = 0$. [2]

(ii) The region R is represented by these three inequalities.

$$2y \geqslant x + 4$$
$$x + y + 2 \geqslant 0$$
$$y \leqslant 2$$

On the grid, shade and label the region R. [2]

(b) Line L is perpendicular to the line $2y = x + 4$.
Line L passes through the point $(1, 8)$.

Show that the equation of line L is $y = 10 - 2x$.

[3]

(c) Use an algebraic method to find the coordinates of the point of intersection of
the lines $2y = x + 4$ and $y = 23 - 2x$.

Answer (.................. , ...) [3]

7(a)(i)	Ruled line from (–4, 2) to (0, –2)	2	**B1** for short or unruled line or for two correct coordinates soi
7(a)(ii)	Correct region shaded	2	**B1** for line $y = 2$ drawn
7(b)	Gradient of $2y = x + 4$ is $\frac{1}{2}$ soi so Gradient of line L is -2	**B1**	
	$8 = -2 \times 1 + c$	**M1**	**FT** substitution of (1, 8) into $y = their\ mx + c$ for L
	Rearrangement to $c = 10$ and hence showing $y = 10 - 2x$	**A1**	
7(c)	(8.4, 6.2) oe from algebra	3	**M1** for a correct method to eliminate one variable **A1** for either $x = 8.4$ or $y = 6.2$ nfww After **A0**, **SC1** for a pair of values that satisfy either equation or for correct answers with no working

8   Find the integers that satisfy $1 < 3x + 5 \leqslant 11$.   m/j18/11/9

Answer   .... [2]

8	−1, 0, 1, 2		2	**M1** for −4/3 < x or x ≤2 or **B1** for 3 correct and none incorrect

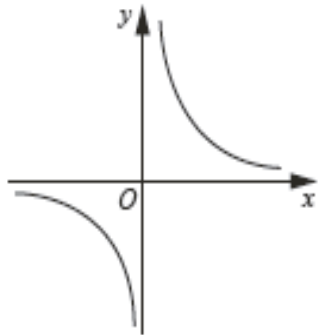9   M/J18/11/23
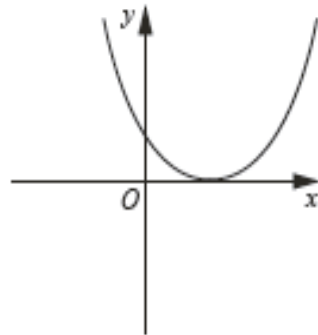

Figure A   Figure B   Figure C
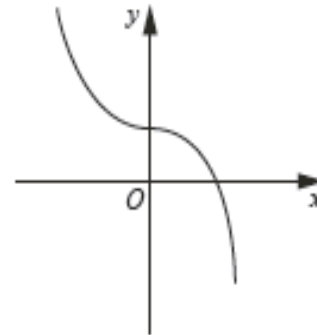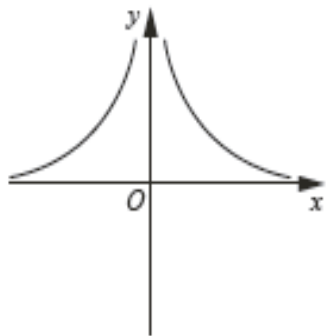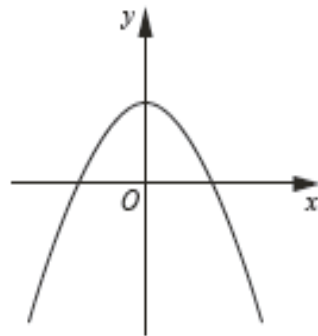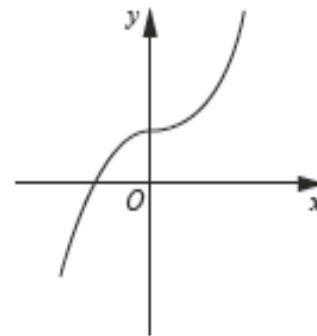
Figure D   Figure E   Figure F

State which of the figures above could be the graph of

(a) $y = x^3 + 2,$

Answer .... [1]

(b) $y = \dfrac{2}{x},$

Answer .... [1]

(c) $y = 2 - x^2.$

Answer .... [1]

9(a)	F	1	
9(b)	A	1	
9(c)	E	1	

10

M/J18/12/18
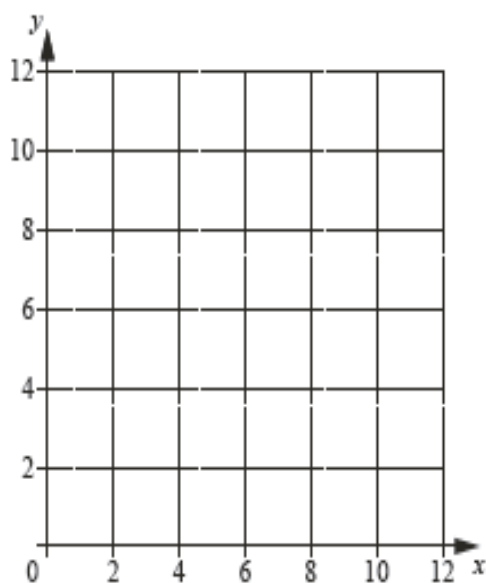
The region R is defined by the inequalities

$$2 \leqslant x \leqslant 8$$

$$5 \leqslant y \leqslant 10$$

$$x + y \geqslant 10.$$

On the diagram, shade and label the region R.   [3]

10	Correct region shaded bounded by $x = 2$, $x = 8$, $y = 5$, $y = 10$ and $x + y = 10$	3	**B1** for line $x + y = 10$ **B1** for at least three correct lines from $x = 2$, $x = 8$, $y = 5$, $y = 10$

11   A is the point $(-4, -1)$, B is the point $(2, 2)$ and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$.   M/J18/22/10

(a) Find the coordinates of the midpoint of AB.

Answer (................... , ...) [1]

(b) Find the gradient of AB.

Answer .... [1]

(c) Show that BC is perpendicular to AB.

[2]

(d) $ABCD$ is a rectangle.

Find the coordinates of point D.

Answer (................... , ...) [2]

(e) Calculate the perimeter of rectangle *ABCD*.

Answer   ....units [4]

.

11(a)	$(-1, \frac{1}{2})$ or $(-1, 0.5)$ cao	1	
11(b)	$\frac{1}{2}$ oe	1	
11(c)	[Gradient of $BC =$] $\frac{-8}{4}$	M1	Alternative 1: **M1** for $\frac{1}{2} \times m_{BC} = -1$ or $m_{BC} = -\frac{1}{0.5}$ oe leading to $m_{BC} = -2$
	$\frac{1}{2} \times \frac{-8}{4} = -1$ hence perpendicular	A1	**A1** for gradient of $BC = \frac{-8}{4} = -2$ hence perpendicular Alternative 2: **M1** for $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ oe and $\overrightarrow{AC} = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$ oe **A1** for $(4^2+8^2) + (6^2+3^2) = (10^2+5^2)$ hence perpendicular
11(d)	$(0, -9)$	2	**B1** for one value correct or **M1** for $\begin{pmatrix} -4 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -8 \end{pmatrix}$ oe or $\begin{pmatrix} 6 \\ -6 \end{pmatrix} + \begin{pmatrix} -6 \\ -3 \end{pmatrix}$ oe
11(e)	31.3 or 31.30…	4	**M3** for $[2\times]\left(\sqrt{3^2 + 6^2} + \sqrt{4^2 + (-8)^2}\right)$ oe or **M2** for $\sqrt{4^2 + (-8)^2}$ oe or $\sqrt{3^2 + 6^2}$ oe or **M1** for $4^2 + (-8)^2$ oe or $3^2 + 6^2$ oe

12

O/N17/11/21
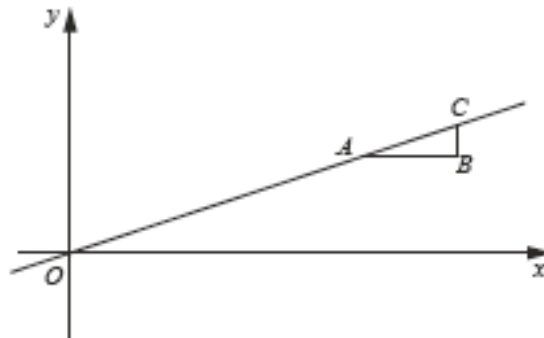
The diagram shows the triangle ABC formed by the lines

$$y = 6, \quad x = 23 \quad \text{and} \quad y = \frac{x}{3}.$$

(a) Find the coordinates of A.

Answer (...................... , ...) [1]

(b) The region inside the triangle is defined by three inequalities.
One of these is $x < 23$.

Write down the other two inequalities.

Answer ....

.... [1]

(c) The point P (h, k), where h and k are integers, lies inside triangle ABC.

Find the values of h and k.

Answer $h =$ ....

$k =$ .... [2]

12(a)	(18, 6)	1	
12(b)	Both $y > 6$ and $y < \dfrac{x}{3}$	1	
12(c)	$h = 22$ and $k = 7$	2	**C1** for one correct

13

O/N17/12/25
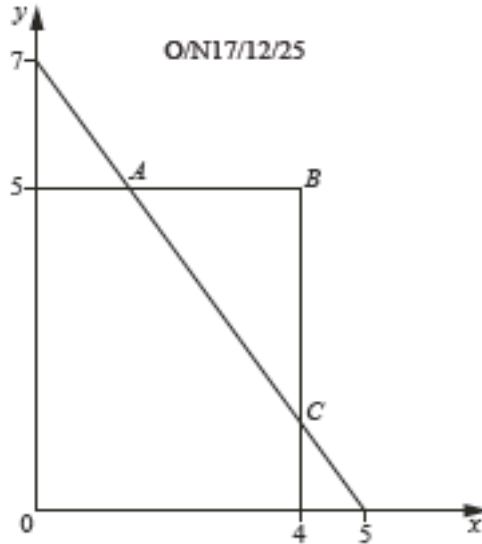
In the diagram, the equation of the line AC is $7x + 5y = 35$.

(a) Write down the three inequalities that define the region inside triangle ABC.

Answer ....

....

.... [2]

(b) The line $y = kx$, where k is an integer, passes through triangle ABC.

Find the greatest possible value of k.

Answer $k =$ .... [2]

13(a)	$7x + 5y > 35$ oe **and** $x < 4$ oe **and** $y < 5$ oe	2	**C1** for two inequalties correct; or for x ... 4 and y ... 5 (with "..." ≠ "< ").
13(b)	3 nfww	2	**B1** for x-coord. of A is $\dfrac{10}{7}$ oe; or for eqn. of OA is $y = \dfrac{7}{2}x$ oe

14   (a)   Write down all the integers that satisfy the $-\dfrac{3}{2} \leqslant x < 2$.

inequality   M/J17/12/9

Answer .... [1]

(b)   Complete the following inequality with a fraction.

$$\frac{3}{4} > \text{.....................} > \frac{1}{2}$$

[1]

(c)   Write down an irrational value of n that satisfies this inequality.

$$2 < n < 3$$

Answer .... [1]

14(a)	−1, 0, 1	1	
14(b)	Correct fraction	1	E.g. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{6}{10}$ etc.
14(c)	Irrational number between 2 and 3	1	E.g. √5, $\frac{2\pi}{3}$ etc.

15 A is the point $(0, 3)$, B is the point $(1, 5)$ and C is the point $(p, -1)$.

(a) Find the equation of the line AB.   M/J17/12/16

Answer .... [2]

(b) The gradient of the line BC is $-\frac{3}{4}$.

Find the value of p.

Answer $p =$ .... [2]

15(a)	$y = 2x + 3$ oe	2	**C1** for $y = 2x + c$ o.e. or $y = mx + 3$ oe $m \neq 0$ or $2x + 3$ or **M1** for gradient = 2 or intercept = 3 soi
15(b)	9	2	**M1** for $\dfrac{5 - -1}{1 - p} = -\dfrac{3}{4}$ oe or for $5 = -\dfrac{3}{4} \times 1 + c$ **and** $-1 = -\dfrac{3}{4} \times p + c$ seen

(d)

M/J17/22/5(d)
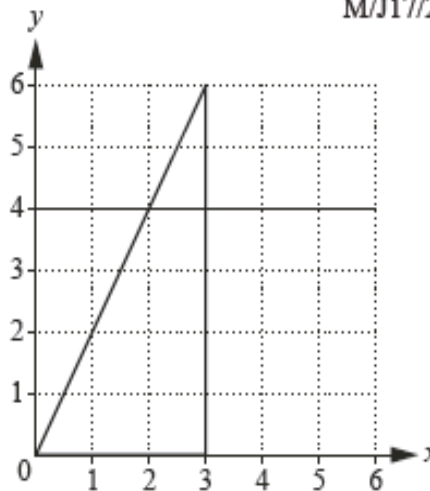
(i) Draw the graph of $x + 2y = 5$. [2]

(ii) Shade the region defined by these inequalities and label it R.

$$x \leqslant 3 \qquad y \leqslant 4 \qquad y \leqslant 2x \qquad x + 2y \geqslant 5$$ [1]

(d)(i)	Ruled line through (0,2.5) and (5, 0)	2	**B1** for 'correct' freehand line or line with a gradient of –0.5 or line through (0, 2.5) with negative gradient or line through (5, 0) with negative gradient
(d)(ii)	Correct region unambiguously identified	1	FT provided *their* straight line with negative gradient and the 3 given lines form a quadrilateral below $y = 4$

16

O/N16/11/24
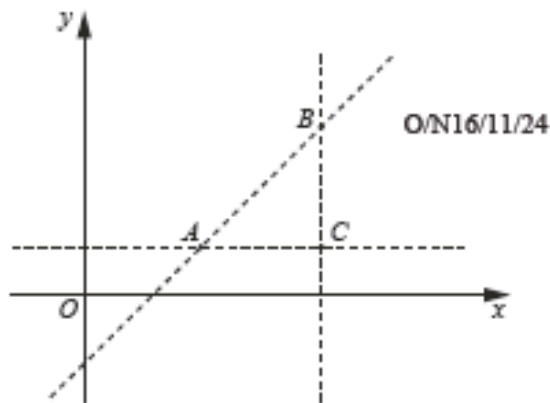
A is the point (5, 2) and B is the point (9, 6).
AC is parallel to the x-axis. CB is parallel to the y-axis.
The equation of the line AB is $x - y = 3$.

(a) Find the coordinates of C.

Answer (.......................... , ...) [1]

(b) The region inside triangle ABC is defined by three inequalities.

Write down these inequalities.

Answer   ....

....

.... [3]

(c) The point (a, b), where a and b are integers, lies inside triangle ABC.

It also lies on the line $y = \frac{1}{2}x$.

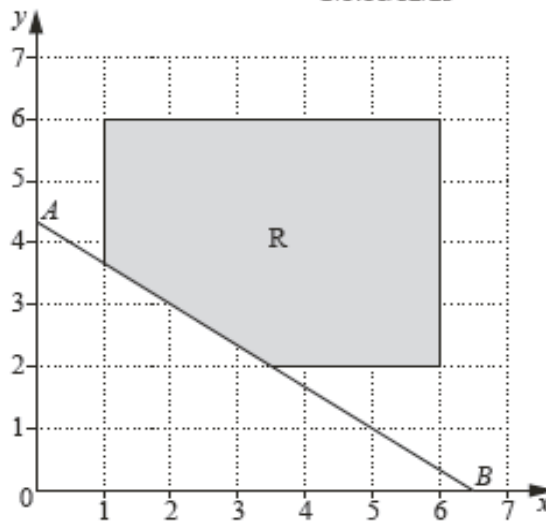Find the value of a and the value of b.

Answer $a =$ ....

$b =$ .... [2]

16 (a)	(9, 2)	1	
(b)	$x < 9$ oe	1	In (b), if [0] scored for $x < 9$ and $y > 2$ then **C1** for both $\{x \dots 9$ or $x \dots$ their(9)$\}$
	$y > 2$ oe	1	and $\{y \dots 2$ or $y \dots$ their(2)$\}$
	$x - y > 3$ oe	1	
(c)	$a = 8$	1	
	$b = 4$	1	In (c), if [0] scored then **C1** for $a = 4$ and $b = 8$; or for $a = 6$ and $b = 3$.

17

O/N16/12/25

In the diagram, the line $3y + 2x = 13$ meets the axes at A and B.

(a) Find the coordinates of A.

Answer (......................... , ...) [1]

(b) The shaded region R is defined by five inequalities.
Two of these are $x \leqslant 6$ and $y \leqslant 6$.

Write down the other three inequalities.

Answer ....

....

.... [2]

(c) The point P is in the shaded region R.

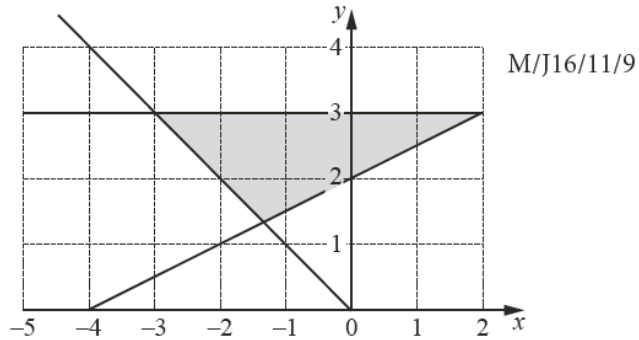Given that AP is as large as possible, write down the coordinates of P.

Answer (......................... , ...) [1]

17 (a)	$(0, 4\frac{1}{3})$	1	
(b)	$x \geqslant 1$ oe, $y \geqslant 2$ oe, $3y + 2x \geqslant 13$ oe – all three	2	**C1** for one or two correct, or for $x \dots 1$ oe, $y \dots 2$ oe, $3y + 2x \dots 13$ oe, with incorrect "...".
(c)	$(6, 2)$	1	

18

M/J16/11/9

The shaded region in the diagram is defined by three inequalities.

One of these is $y \geqslant \frac{1}{2}x + 2$.

Write down the other two inequalities.

Answer ....

.... [2]

18	$y \leqslant 3$ oe	1	C1 for y ... 3 oe and y ... $-x$ oe, where
	$y \geqslant -x$ oe	1	'...' is the wrong inequality or =

19 The coordinates of the midpoint of the line AB are $(1, 2)$.
   The length of the line AB is 10 units.   M/J16/22/14

   (a) If the gradient of AB is 0, find the coordinates of A and B.

                                   Answer $A = ($.............. , ...$)$

                                           $B = ($.............. , ...$)$ [1]

   (b) If the gradient of AB is $\frac{3}{4}$, find the coordinates of A and B.

                                   Answer $A = ($.............. , ...$)$

                                           $B = ($.............. , ...$)$ [2]

20   The diagram shows the lines $x + y = 8$ and $2y = x + 4$.

M/J16/12/15
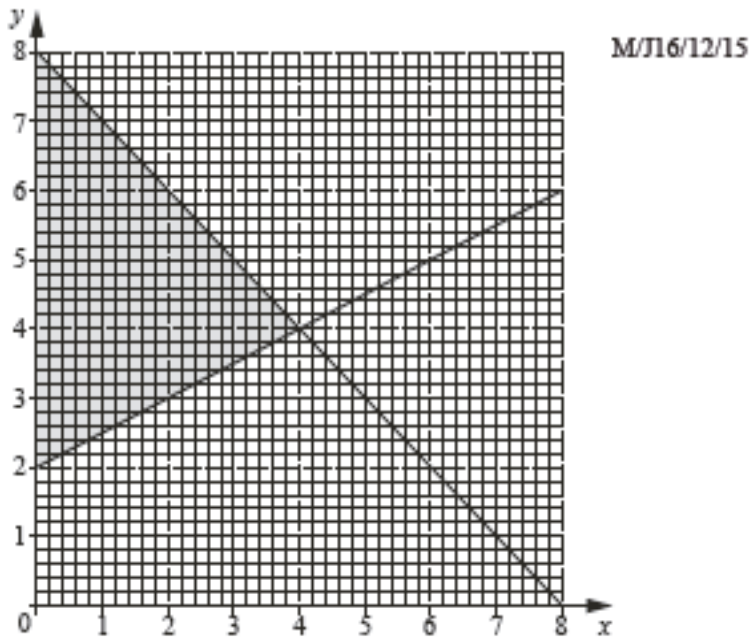
(a)   The shaded region on the diagram is defined by three inequalities.

Write down these three inequalities.

Answer   ....

....

.... [2]

(b)   Another region, R, is defined by the inequalities $x + y \leqslant 8$, $2y \leqslant x + 4$ and $y \geqslant a$,
where a is an integer.
This region contains 5 points with integer coordinates.

Write down the value of a.

Answer $a =$ .... [1]

19	(a)	(−4, 2) (6, 2)	1	Both correct
	(b)	(−3, −1) (5, 5)	2	**C1** for one correct or for two x-values or two y-values correct or for **both** (4, 6) **and** (−2, −2)
20	(a)	$x + y \leqslant 8$ oe $2y \geqslant x + 4$ oe $x \geqslant 0$	2	**C1** for two correct
	(b)	3	1	